On the Metaphysics of Quantum Mechanics: Why the Wave Function is not a Field

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1 Introduction

If we were to be realist about quantum mechanics and regard the wave function as real, what physical object would the wave function be? There is a famous proposal called wave function realism according to which (i) the wave function is a real physical field living in configuration space and (ii) configuration space is the fundamental physical space. Because the mathematical object representing the wave function is naturally defined on configuration space, the metaphysical content of claim (i) hinges heavily on claim (ii). At the same time, claim (ii) is highly controversial: the number of dimensions of the configuration space of the universe is close to 10^{80} on current best estimates (if we only restrict ourselves to baryons) and this presumably leaves us with a preposterous metaphysics for quantum mechanics. The main competing proposal, the so-called primitive ontology approach, rejects in block any commitment to a physically real configuration space and typically denies the claim (i) likewise. It is defended instead that the metaphysics of quantum mechanics is based on the assignment of properties or objects to the three-dimensional space in which we live.

If we are to evaluate claim (i), we therefore need to look more closely at claim (ii). The problem is that both camps do not seem to share the same standards for evaluating what a fundamental physical space is supposed to be. The first and main task of this paper is thus to clarify the source of the disagreement, and I introduce for that purpose four necessary and sufficient conditions for a space to be physically fundamental. I show that the most crucial issue for wave function realism is not the connection to the three-dimensional manifest image, as it has been widely argued, but the violation of what I call the condition of 'dynamical matching': namely, that the symmetries of the fundamental space must match the symmetries of the fundamental dynamics. The primitive ontology approach does not fare better however, since it violates what I call the condition of 'physical completeness': namely, that the fundamental space must embed the complete history of fundamental facts. On the face of it then, neither the (standard) wave function realism nor the primitive ontology proposal provides us with a fundamental physical space. I do not claim, however, that the debate cannot be settled by either providing reasons for relaxing one or some of the conditions I offer, or arguing for the best space with the help of complementary conditions.

Either way, the argument of this paper will be more modest. I restrict myself to the claim that, for each of the two best candidates for a physically fundamental configuration space, the wave function does not qualify as a *physical* field in the standard sense (and hence that the widespread analogy with classical electromagnetism in the literature is misguided). I leave open the possibility of another candidate-ontology and conclude by offering a series of appetizers supposed to make us even more skeptical about the field-status of the wave function.

The paper goes as follows. Section 2 rehearses some of the basics of quantum mechanics and the main motivations for taking the wave function realism and the primitive ontology proposals seriously. Section 3 outlines and motivates the four conditions for a space to be physically fundamental. Section 4 explains why the wave function realism and primitive ontology proposals do not provide good enough space-candidates and I offer at the occasion complementary conditions which we have good reasons to endorse given the physical spaces displayed by our best theories so far. Section 5 argues that for the two best configuration space candidates for wave function realism, the wave function does not qualify as a physical field in the standard sense.

2 Quantum mechanics and wave function realism

One of the basic ideas of the standard formulation of quantum mechanics is that all the features of a system at a given time are *fully* encoded in a state vector $|\psi(t)\rangle$ defined in an abstract Hilbert space $\mathcal{H}$. As such, the state vector is rather physically uninformative. The physical features of the system are specified by associating the state vector with observable properties — for instance, the momentum property represented by the operator P and the corresponding vector basis $\{|p\rangle\}$, each vector in this basis defining a possible momentum state of the system.[1] Once all the information needed is gathered, the state of affairs of the system at a certain time can be fully characterized by evaluating $|\psi(t)\rangle$ on a complete basis accounting for all the independent properties of the system. If we leave aside spin and other properties for simplicity, the position basis $\{|x\rangle\}$ gives the most common and physically intuitive representation of the state vector: we define the "wave function" as the first fundamental ingredient of the theory by projecting the state vector onto the position vectors $\psi(x,t) := \langle\psi(t)|x\rangle$, and the space over which the wave function is defined is called "configuration space". This space is $3N$-dimensional for a N-particle system evolving in three dimensional space $\psi(x_1, ..., x_{3N}, t)$.

The second fundamental ingredient of the standard formulation is that the state vector, or wave function, evolves in time according to a unitary, deterministic and linear dynami-

[1] More technically, all Hilbert spaces with the same dimension are isomorphic. We therefore need some structure on the Hilbert space to characterize a specific system. Usually, this amounts to defining a preferred set of observables (or a preferred set of basis vectors) and a specific decomposition of the system into subsystems if needed.

cal equation.[2] The state vector defined in the Hilbert space together with the complete set of relevant observables and the dynamical equation is usually called the "bare quantum formalism". But that is not all. In the standard formulation, the bare quantum formalism is equipped with an additional interpreted algorithm for relating the state vector to measurement outcomes: the Born rule specifies that the probability density for the system to be found at some position x at some time t is equal to the absolute square value of the wave function $|\psi(x,t)|^2$, and the collapse postulate states that the system evolves upon measurement into some specific state $\psi(x,t)$ in accordance with the Born rule.

Now these ingredients taken together lead to the infamous measurement problem. Traditionally, it has been expressed as the fact that the dynamical equation is incompatible with the collapse postulate.[3] Suppose that we pick for instance a toy-model particle in the superposition state $|\psi\rangle_p = \frac{1}{\sqrt{2}}(|x_1\rangle + |x_2\rangle)$ and decide to measure its position with some pointer measurement device initially at rest (hence in the state $|\text{"rest"}\rangle_m$). Then, the contradiction arises from the fact that the dynamical equation predicts that the system made of the particle and the pointer evolves into the entangled state $1/\sqrt{2}(|\text{"}x_1\text{"}\rangle|x_1\rangle + |\text{"}x_2\text{"}\rangle|x_2\rangle)$ while the collapse postulate (together with the Born rule) predicts that the system will be either in the state $|\text{"}x_1\text{"}\rangle|x_1\rangle$ or in the state $|\text{"}x_2\text{"}\rangle|x_2\rangle$ with probability $1/2$ in each case. More generally, we can think of the measurement problem as the fact that the standard formulation does not specify enough the notion of "measurement" for the formalism to be consistent with our ordinary experience of definite measurement outcomes.

The measurement problem has been one of the most preoccupying difficulties plaguing the foundations of quantum mechanics since the 1930s or so, and there are currently several promising proposals in vogue — for instance, the so-called Dynamical Collapse (e.g. GRW), Bohm-de Broglie hidden variables, Everett, Modal and consistent-histories interpretations of quantum mechanics. These proposals can be classified on the basis of whether they modify the bare quantum formalism. The Everett interpretation, for instance, leaves the formalism untouched and provides a realistic interpretation of superpositions as full-fledged multiplicities (e.g. many emergent worlds). Some of the other solutions to the measurement problem are based on a radical modification of the dynamical equation. The GRW theory for instance introduces an irreducibly stochastic element in the evolution of the state vector, while Bohmian mechanics supplements the dynamical equation of the wave function with a guiding equation determining the evolution of hidden variables. There are of course various ways of concocting and interpreting these theories. They are *also* various ways of formulating the measurement problem to incline others towards one's favorite interpretation. But, as far as it goes, none of the solutions to the measurement problem has been considered to be free from difficulties, nor has any of these solutions been considered overwhelmingly successful for a consensus to emerge.

The measurement problem deserves the widespread attention it has received but it

[2]This means (respectively) that the total sum of the probabilities for the system to be found in each possible state is conserved through time; that the evolution of the system is uniquely specified given initial conditions; and that, given two state vector solutions to this dynamical equation, the sum of these state vectors with some arbitrary coefficient is also a solution (this is coined a "superposition" of state vectors).

[3]No surprise: the collapse postulate defines an indeterministic, non-linear and non-unitary evolution of the wave function into a specific state upon measurement.

should not be seen as blocking any attempt to understand further the philosophical implications of quantum theory. Leaving the question of locality aside (among others), the question of the ontology of the wave function has received increasing attention since 30 years or so, partly because it became clear that most of the serious solutions to the measurement problem were underwriting a realistic stance towards the wave function. That is, it became clear that the instrumentalist interpretation inherited from Bohr (among others) and its modern variants were deeply unsatisfying.[4] In the early orthodox interpretation, the wave function was considered, to quote Bell, "simply as a convenient but inessential mathematical device for formulating correlations between experimental procedures [state preparation] and experimental results [state measurement]" (1987, p. 53). The ontology of the wave function, however, is not a brand-new topic, witness Einstein and Schrödinger's early writings, but it now appears to be a question worth asking.[5]

There are two main realistic approaches to the interpretation of the wave function currently in vogue. (i) On the so-called wave function realism approach, at least as it was initially formulated, the wave function is considered as a fundamental physical field living in configuration space (e.g. Bell, 1987, pp. 128, 134, 164, 204; Albert, 1996, p. 278). This proposal contains in fact four implicit claims, laid out here by order of dependence:[6]

(a) The wave function is physically real (i.e., strictly speaking, 'wave function realism')

(b) Configuration space is physically real ('configuration space realism')

(c) The wave function is (moreover) a physical field ('wave function field realism')

(d) The wave function and configuration space are (moreover) fundamental (I will call this 'wave function fundamentalism')

The claim (d), as I put it, leaves open the possibility of being reductive or pluralist about the typical three (or four) dimensional space and its objects. In what follows, I will ignore any complaint about the claim (a) and any debate about the claim (d). One extreme form of (d) is the so-called wave function monism view, according to which all there is about a quantum system is its wave function.

The full ambition of the wave function realism proposal only emerges once we remind ourselves that all the quantum theories we have so far formally describe multi-component systems, whether we take the components to be particles, fields, geometries or any other exotic quantized object. For instance, the Schrödinger equation for a system with $3N$ mechanical degrees of freedom is defined as:

$$i\hbar\frac{\partial\psi}{\partial t}(x_1,...,x_{3N},t)=\sum_{i=1}^{3N}\frac{-\hbar^2}{2m}\nabla_i^2\psi(x_1,...,x_{3N},t)+V\psi(x_1,...,x_{3N},t) \tag{1}$$

where $\hbar$ is a constant, m the mass of the system and V the potential or interaction term. In what follows, I will keep things simple and follow the literature on wave function realism by

[4]About the contemporary neo-Copenhaguen interpretations of quantum mechanics, how they solve the measurement problem, and what sort of issues they lead us into, see Wallace (2008, sec. 3).

[5]E.g. see the series of papers "Quantization as a problem of proper values" in Schrödinger (1928).

[6]I thus disagree with Myrvold who only detects three underlying claims (2015, p. 3250). For the coining "wave function field realism", see Solé and Hoefer (2015, p. 4).

adopting the analogy with the classical Hamiltonian mechanics of N particles evolving in three dimensional space, according to which the wave function $\psi(x_1, ..., x_{3N}, t)$ is governed by the typical Hamiltonian:

$$H = \sum_{i=1}^{3N} (p_i)^2 + \sum_{i,j=1;\ i<j}^{N} V_{ij}((x_{3i-2} - x_{3j-2})^2 + (x_{3i-1} - x_{3j-1})^2 + (x_{3i} - x_{3j})^2) \qquad (2)$$

In the general case, we need to define the wave function of a multi-particle system in terms of generalized coordinates $\psi(q_1, ..., q_{3N}, t)$; the wave functional of a multi-field system in terms of field configurations $\Psi(\phi(x), g_{\mu\nu}(x), ...)$; and so on and so forth. The striking fact shared by all these cases is that, on the wave function realism proposal, the universal wave function lives in a very high dimensional physical space — 10^{80} for the estimated number of baryons in the universe, and infinite for quantum field theories well-defined on the continuum (!).

No doubt this is a madcap story. But there is a deep motivation for taking the 'configuration space realism' claim seriously. The wave function is a separable object in configuration space, which means that any property of the wave function defined over some region R is completely determined by its properties defined over all the sub-regions of R. Hence, we can provide (in principle) a complete physical account of the system of interest by specifying the properties of the wave function at each point of configuration space. By contrast, the assignment of quantum mechanical properties of the system in the three-dimensional space leaves physical information out — say, the specific type of correlations between entangled sub-systems. The situation is thus completely new compared to the N-particle classical case for which configuration space and the three dimensional space are physically equivalent. That we *need* to describe quantum systems with a mathematical function on configuration space is even a defining feature of quantum mechanics, insofar as we know that sub-atomic processes have mostly indefinite properties and are mostly entangled with one another.

(ii) The primitive ontology approach, on the other hand, assigns the fundamental properties of the quantum system to regions of the typical three dimensional space (or four dimensional space-time).[7] The three dimensional space thus contains non-separable facts, but non-separability is not taken as a determining criterion for deciding upon the ontology of quantum mechanics.[8] What motivates primarily the primitive ontology approach is that the fundamental ontology of a theory should be expressed in terms of "local beables", that is, objective three (or four) dimensional objects whose properties connect straightforwardly to the typical observational features of our experience.[9] If the theory were to deny the existence of three-dimensional objects, then it would fall short of the things which underwrite our capacity to measure, test and infer from empirical evidence. In a word, this theory would deny the existence of the very means by which it can be said to be empirically adequate — it would simply be empirically incoherent. The typical

[7] See Goldstein (1998) for the coining "primitive ontology".

[8] See for instance Maudlin's insistence that we should distinguish between informational and ontological completeness (2007).

[9] See Bell (1987) for the coining "local beables". See Maudlin (2007) for further details.

primitive ontologist thus denies that the high dimensional configuration space is real in any fundamental sense. But this does not announce the return of the instrumentalist orthodoxy: the wave function is real in the sense that it governs the evolution, or encodes the dispositional properties, or determines the relations of local beables. That is, the primitive ontology approach is typically dualistic with the proviso that the fundamental objects of quantum mechanics are three-dimensional.

There are, of course, various proposals for what the wave function and the local beables are supposed to consist of depending on one's preferred solution the measurement problem. To give a single example, the typical wave function realism approach to Bohmian mechanics takes the evolution of what we take to be N particles in three-dimensional space to be determined by a unique universal point-particle living in $3N$ configuration space and guided by the wave function. On the typical primitive ontology approach, the local beables are defined by N real particles and the wave function is thought of as a guiding law determining their motion (e.g., Goldstein and Zanghi, 2013, pp. 92-3).[10]

3 What it means to be a fundamental physical space

The debate over wave function realism as it has been carried out so far is rather surprising. There seems to be a lack of clear and minimally agreed-upon standards for assessing wave function realism over the primitive ontology approach, whether we speak of what it means to be a fundamental space or what the ontology of a physical theory consists of — the result being that the two approaches fail to be on the same wavelength. Configuration space realism is of course the most crucial, revisionary and accordingly controversial claim of the wave function realism proposal on which much of the debate hinges. Wave function realists emphasize that what makes a space physically fundamental is its ability to host the complete set of fundamental facts, while primitive ontologists argue that the best candidate for the fundamental space is the least revisionary one if the corresponding theory is empirically and explanatory adequate (and that, in most cases, the three or four dimensional space is the best fit). I offer in this section four sufficient and necessary conditions for a space to be physically fundamental as a minimal agreement for the debate to go on (I will call this the "minimal model"). To make the discussion more concrete, I will explain what these conditions amount to if we were to take classical Hamiltonian mechanics (CM) to be a fundamental theory and the corresponding "Galilean" or "neo-Newtonian" space-time to be a fundamental space.[11]

(1) <u>The minimal extensive structure</u>: A physically fundamental space has a minimal structure of extension, i.e. it defines a relation of "closeness" between elements.

<u>CM example</u>: Space-time is typically defined by a four dimensional Euclidean space with a specific temporal foliation defining a series of three dimensional Euclidean

[10] Note the interesting fact that if the wave function is considered to be a guiding "force", it is not a Newtonian force but rather an Aristotelian one insofar as it produces velocities rather than acceleration (Brown, 1999, p. 56).

[11] Note that these conditions can be both formulated from a substantivalist or relationist standpoint (although here, "fundamental" suggests that we should favor the former).

spaces. The temporal and spatial Euclidean metrics are independent.

What makes the condition (1) necessary? The intuition is that a physical space is the host of dynamical happenings, that is, of anything like actions and interactions which *take place*. But for anything to take place over distinct locations, there must exist distinct elements and a minimal notion of what it means for them to be "close" to one another. In a word, dynamical happenings demand a minimal structure of extension. The specific form of that minimal structure is controversial, however.[12] (i) It is likely that we need at least a topological space, that is, a structure constituted of elements and a neighborhood relation between these elements. For instance, the set of three points $X = \{1, 2, 3\}$ together with the topology defined by the collection $T = \{\{\}, \{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$ defines the neighborhood of the element 1, and hence what it means for the other elements to be "close" to 1.[13] (ii) We might also need a quantitative notion of "being close to" (i.e. define a notion of "distance") and therefore use a metric space.[14] (iii) Most likely, although this is controversial in quantum gravity and cosmology, the fundamental space has the minimal structure of a smooth (i.e. infinitely differentiable), connected manifold M with a metric g_{ab}. This naturally defines notions of continuous extension, straightness, connected path, location, displacement and degrees of displacement.[15]

(2) Physical completeness: The complete set of fundamental, irreducible physical facts is determined by specifying properties at each distinct spatial element of the fundamental space.

CM example: We can determine all the properties of objects — velocities, accelerations, masses, etc. — by specifying their positions across time.

The intuition behind this condition is that if the physical space is fundamental, then it does not leave out fundamental physical facts and thus make the use of additional structure dispensable (either another space or irreducible fundamental objects). Most crucially, this condition ensures that the fundamental space is not the host of non-local or non-separable objects (cf. Sec. 2). This is precisely the sort of argument according to which space is supposed to be non-fundamental in string theory, or at least not a primitive concept, since strings (and other exotic objects) are non-local objects (e.g. Gross, 2005, p. 105). This is also the condition on which wave function realists bet the ranch: configuration space is precisely the space "in which a specification of the local conditions at every address at some particular time (but not at any proper subset of them) amounts to a complete

[12] Obviously this minimal structure needs to be expressible in a mathematical form. This excludes in particular fictional spaces.

[13] The spatial elements do not need to be points however, although points (strictly speaking) surely figure as the most natural candidates given that they are identical, structureless simples. About the problems related to point-geometries in quantum mechanics, cf. Arntzenius (2003).

[14] Note that having a metric does not mean that the topology is fully fixed. For instance, the infinite Euclidean two-dimensional plane and the Euclidean two-dimensional cylinder have the same metric but different global topologies.

[15] Note that if the (most) fundamental space were to emerge out of more primitive fundamental objects as suggested by the recent developments in quantum gravity, these fundamental objects would need to be themselves somehow "extended" for any self-interaction or cross-interaction to be possible.

specification of the physical situation of the world, on that theory, at that time" (Albert, 1996, p. 282).

(3) Dynamical matching: The complete set of fundamental, irreducible physical facts has a unique physical order on the fundamental space.

CM example: The fundamental equations define unique solutions, i.e. unique ordering of physical facts, up to transformations under the Galilean group $Gal(3,1)$, which matches the natural symmetry structure of the Galilean space-time defined by its metric.

The intuition here is that a physical space cannot be fundamental if the fundamental physical facts are arranged into an (i) arbitrary and (ii) redundant mosaic on this space. First, the space must be such that the fundamental facts follow a regular causal physical order, i.e. that the physical relations enforced by the dynamical equations are instantiated on the space. If this is the case, the fundamental space defines a well-ordered history and geography of facts — or, to use Albert's expression, an "arena" (1996, p. 282; 2013). We are thus preventing that unconnected physical events pop out randomly as the history of the world unfolds.[16] But that is not enough. The order must also be physically unique: namely, that each physical fact is assigned to only one spatial element and that there are no "idle" spatial elements or redundant physical facts. For that, we must enforce the fundamental symmetry principle that there must be as much symmetry in the space as in the dynamics (more about this in Sec. 4).[17]

(4) Empirical adequacy: The fundamental space is empirically adequate, i.e. observable data can be consistently embedded within the theory in which the fundamental space is defined.

CM example: Measurements can be registered by the successive positions of three dimensional material bodies.

This condition relies on the fact that the existence of the fundamental space must be compatible with us gathering empirical evidence for having good reasons to believe in its existence. The most straightforward way of satisfying this condition is that the fundamental space reproduces or connects to the three dimensional space whereby we make observations, or at least explain why we are justified in believing in the empirical data that are registered by what appears to us as three dimensional objects. There are several ways to define the relation between the fundamental space and our typical three-dimensional space — this is part of the so-called "macro-object problem", that is, the challenge of providing "truth conditions for assignments of ordinary properties of macro-objects" given the wave function in configuration space (Belot, 2012, p. 2; see also Ney, 2016). The best

[16] But note that this does not prevent non-local effects if they are permitted by the dynamical equations, or that the dynamical equations are stochastic.

[17] It does not mean that all the space-time symmetries are necessarily fixed *a priori*, nor that we cannot use physically equivalent descriptions of the ordering. A symmetry principle only defines a relation of coherence between the dynamical facts and their spatial assignment — and obviously, the character of the specific symmetries at play depends on the nature of the space and of the dynamical facts.

is to assume that the physical facts defined on the fundamental space must provide a sufficient supervenience basis for the emergence of three dimensional phenomena (e.g. Wallace and Timpson, 2010, p. 705). But this is not uncontroversial, and relatively vague since there are distinct types of supervenience relation (more about this in Sec. 4). Be that as it may, the condition of empirical adequacy requires at least that the fundamental space determines the existence of one stable global three dimensional pattern.[18]

One might object that the conditions (1)-(4), though fulfilled by the typical spaces defined by our best theories, are too constraining for the sort of fundamental physical space we might *conceive*. We can conceive for instance that the fundamental physical space just takes the form of a single point p and does not include any minimal structure of extension (and hence any sort of "spatio-temporal relations"). We can conceive furthermore that all the elements of the complete and presumably infinite list of fundamental dynamical facts are assigned to this point by some mapping $\{a_1(p), a_2(p), ...\}$. But this means that we need to consider dynamical relations as facts too, for otherwise it is unclear how facts could relate to one another in any way— for instance, an element $a_3(p) = T_5(a_1(p), a_2(p))$ that states that the fact a_1 happens five minutes before the fact a_2. But if we do take dynamical relations as facts and not as spatio-temporal properties, there is no reason why they should not be related to one another too — say the "fact" that five minutes is a shorter duration than ten minutes, and so on and so forth. But all that this toy-example seems to reproduce is exactly a well-ordered structure of extension relating dynamical facts, with the additional metaphysical commitment to spatio-temporal facts. The merit of this example is to suggest that dynamical facts cannot be related unambiguously without a minimal structure defining spatio-temporal relations, although it does not mean that the minimal structure needs to be a manifold.

What about the relation between (2), (3) and (4)? The previous example suggests that, in order to provide a complete assignment of fundamental physical facts, there must be a unique ordering. In the case of multiple orderings, either nothing decides upon the correct assignment and its remains ambiguous, or we must assign several facts to one spatial element and the assignment is redundant. Likewise, there cannot be any correct ordering — or at least any fundamental ordering — if only a partial set of fundamental facts is assigned to the fundamental space. On the face of it, the conditions (2) and (3) seem to be at the same level. The last condition (4) is also necessary but less fundamental for two reasons. The first is that nothing seems to prevent the empirical evidence for the fundamental space to be extremely loose. The second is that our three-dimensional experience and our belief in empirical data could be completely illusory. Yet, these two reasons do not make the condition (4) superfluous. We must be able to explain at least why the fundamental space makes it difficult for us to gather evidence in its favor or why the fundamental space gives rise to the illusion that the actual world in which we live is three dimensional.

Obviously nothing that I have argued in this section proves that there is no knock-down argument against the existence of a fundamental physical space that would violate

[18]That is, on the minimal account, the condition of empirical adequacy does not prevent the existence of distinct three dimensional spaces or of other intermediary less fundamental spaces.

one of these conditions. Nor have I shown that these four conditions are sufficient. But the discussion above still makes it reasonable that these four conditions are essential to the definition of a physically fundamental space.

4 There's the rub

We may now address the question of whether configuration space realism and three-dimensional realism meet these four conditions.

Much attention has been paid so far to the issue of the explanatory gap between the recovery of three dimensional appearances and the behavior of the wave function in configuration space; that is, in our language, to the idea that wave function realism presumably fails to meet the condition of empirical adequacy (e.g. Lewis, 2004; Monton, 2006; Maudlin, 2007; Ney, 2012). For instance, Monton (2002, 2006) has argued that a point in configuration space does not specify by itself a single arrangement of N particles. The axes of configuration space could be switched or the origin of one coordinate axis shifted: in both cases, the transformation would yield distinct three dimensional N-particle configurations.

Consider for instance the case where the wave function governed by Eq. (2) is six-dimensional. Suppose furthermore that it is non-zero at the point $(1, 0, 0, 1, 0, 0)$ defined in some Cartesian coordinate system $(x_1, x_2, x_3, x_4, x_5, x_6)$. If we switch the axes x_1 and x_2, the wave function is non-zero at the point $(0, 1, 0, 1, 0, 0)$ in the new coordinate system. But the two descriptions are physically equivalent since the choice of coordinate system is arbitrary — for instance, all the distances in the six dimensional space are preserved when we change the coordinate system. The situation in the three dimensional space is different, however. If we consider for instance that (x_1, x_2, x_3) and (x_4, x_5, x_6) are the (x, y, z) coordinates of two particles, the first configuration in the six dimensional space implies that the two particles are at the same location while the second does not. According to Monton, we thus need to add to the description a correspondence principle between the coordinate system in the $3N$ dimensional space and the particle configurations in the three dimensional space so that the three dimensional appearances are unambiguously defined from the wave function.

Lewis (2004, 2013) provides a straightforward reply which, I think, is standard among the proponents of wave function realism (e.g. Albert, 1996; Ney, 2012; North, 2013). It is the dynamical evolution of the wave function and not simply its synchronic state that determines the three dimensional particle-configurations. What we even mean by 'particle', their number, the way they are arranged together, the way they are labeled and identified by a coordinate system — all of these features emerge from the structural relations defined by the Hamiltonian in Eq. (2). For instance, particles are not only defined by the highest localized peaks of the wave function but also by the way these localized peaks evolve and remain relatively stable under temporal evolution. Following Albert (1996), the three dimensional space simply corresponds to the space of interacting distances as instantiated by the $3N$ degrees of freedom of the wave function.

There are, to be sure, some issues about *interpreting* how the wave function living in configuration space connects to three dimensional phenomena: namely, about (a) providing the adequate reading of the dynamical equations, (b) specifying the correct type of dependence relation (e.g. nomic supervenience, functional reduction, reductive vs. eliminative), (c) adjusting the interpretation in the light of one's specific solution to the measurement problem, (d) evaluating whether the emergent three dimensional space is rather relational or substantival, and so on and so forth.[19] While these interpretative matters are crucial to the *explanation* of our three-dimensional experience, they do not threaten the empirical adequacy of the fundamental space. It is sufficient that the wave function defined on configuration space possesses a substructure isomorphic to a three dimensional structure for the condition (4) to obtain.

The real trouble for wave function realism rather comes from the dynamical matching condition. The natural symmetries of a space are given by its metric: given the $3N$ dimensional Euclidean metric, the natural physical symmetry group of the $3N+1$ dimensional space-time in the non-relativistic setting is $Gal(3N, 1)$. This includes rotations, spatial and temporal translations, and boosts defined by $9N+1$ parameters. Let us call this space embedded with this structure the "bare configuration space" $(\mathbb{R}^{3N}, g_{3N})$. But note that the dynamics of the wave function in Eq. (2) is only covariant under the symmetry group $Gal(3, 1)$ defined by 10 parameters.[20] The standard account of wave function realism therefore violates the well-endorsed symmetry principle that, for a space to be precisely considered *physical*, it must be sufficiently structured for the formulation of the dynamics but not more (e.g., Earman, 1989, pp. 41-8; 2004, sec. 4; Brown, 2005).[21]

More precisely, this symmetry principle has two facets according to Earman. First, (SP1) requires that any dynamical symmetry is a space-time symmetry. If this fails, then some structure of the fundamental space fails to have a dynamical counterpart — for instance, some fact about spatial locations in the case where the dynamics is invariant under spatial translation. At first sight, by a simple application of Ockham's Razor, there does not seem to be good reasons to commit to the idle spatial structure, and we should favor the dynamically equivalent fundamental space where no such spatial structure is present. But this is only a weak formulation of the potential threat. More generally, the existence of idle spatial structure often signals that the assignment of physical facts on

[19] See Ney (2016) for a summary of several proposals. See also Lewis (2004); Maudlin (2007); Albert (2015).

[20] There is actually much to say about symmetry transformations. For conciseness, let me put some important details here. (i) if we leave time aside, the natural geometrical structure of the bare configuration space is the Euclidean group $Eucl(3N)$ (that is, the group of isometries $ISO(3N)$ on $\mathbb{R}^{3N}$, and the mismatch is thus between $Eucl(3N)$ and $Eucl(3)$). (ii) The Galilean group $Gal(3N, 1)$ is defined by four types of transformations: translations in time ($t' = t + a$) and rotations, translations and boosts in the $3N$ dimensional space ($x' = Rx + vt + b$), where a is the temporal translation parameter, R the rotational matrix, v the Galilei boost parameters and b the spatial translation parameters. $Gal(3, 1)$ is thus a subgroup of $Gal(3N, 1)$ for $N > 1$ (for a master survey of Galilean symmetries in the context of quantum mechanics, cf. Lévy-Leblond (1971)). (iii) A covariant symmetry transformation of the dynamics is a transformation that leaves the form of the dynamical equations invariant but not necessarily the dynamical and non-dynamical objects of the equations (for the difference between covariance and invariance, see Brading and Castellani (2005, sec. 4.1).). (iv) The form of the dynamical equations is actually invariant under a $3N$ dimensional reducible representation of $Gal(3, 1)$. It means that the corresponding Galilean matrix transforming position vectors has N identical blocks.

[21] Cf. Wallace and Timpson (2010, pp. 700-1) for the remark in the context of wave function realism. The violation of the symmetry principle also occurs for the wave functional equation.

the space is underdeterminated, i.e. the physical ordering of facts is not unique (recall the condition of dynamical matching in Sec. 3).[22] This will be the case for wave function realism (cf. below).

That being said, it seems that (SP1) has at first sight many counter-examples: for instance, the gauge symmetry of the vector potential in classical relativistic electromagnetism is a symmetry of the dynamical equations but supposedly not a symmetry of space-time. In the quantum field case, the gauge symmetry transformation applies on the internal degrees of freedom of the matter fields and not on their "external" spatio-temporal ones. Discrete symmetries provide another striking example: typical wave functions are either antisymmetric or symmetric under the permutation group and these transformations leave the dynamical equations invariant. But these are not spatio-temporal symmetries strictly speaking.

These sorts of counter-examples are actually misleading — not because they are examples of spatio-temporal symmetries but because they conflate an important aspect: namely, that we must distinguish between the symmetries of the objects and the symmetries of the dynamical equations (properly understood).[23] On the one hand, the symmetry of an object does not need to be compatible with the physical transformations allowed by the dynamical equations, i.e. with the possible ways dynamical objects can evolve. For instance, the permutation of two (three-dimensional) coordinates of the wave function can be interpreted on the active reading as the transformation that swipes two sets of facts (e.g. two particles) on configuration space, but this transformation does not have any dynamical counterpart in the standard Hamiltonian. On the other hand, dynamical symmetries always have an active interpretation: they correspond to the various ways physical objects can evolve and which eventually leave the physical situation invariant. For instance, a velocity boost can be brought about by accelerating the system during a short amount of time. But once this is done, the dynamical equations have exactly the same form as before the boost. Now, since the dynamical facts are spatio-temporal, (SP1) must only capture symmetries which correspond to dynamical transformations. If this is the case then, (SP1) does not seem to have any direct counterexample.

Second, (SP2) requires that any space-time symmetry is a dynamical symmetry. There are two good reasons underlying this principle. First, the dynamical laws, insofar as they are supposed to be universal, need to be physically equivalent at different locations on the space. For otherwise, it means that the structure of the space itself is dynamically responsible for the way the laws vary and therefore that the dynamical equations leave physical facts out (i.e. we need to include the effect of the spatial structure into what we initially took to be universal laws). Second, the fundamental laws, insofar as they are supposed to be *objective*, should be independent of the coordinate system in which they are

[22]Note that the lack of a unique physical ordering can take the specific form of a failure of determinism, but I take the former to be more general.

[23]This distinction seems to match Earman's characterization of dynamical symmetries. According to Earman (1989), a classical theory of motion is defined as a class of models $\langle M, A_1, A_2, ..., M_1, M_2 \rangle$ where M is a space-time manifold, the A_i are absolute objects characterizing the space-time structure (e.g. an absolute referential frame), and the P_i are dynamical objects characterizing the physical content on this space-time. The dynamical symmetries are diffeomorphisms that drag the dynamical objects on the space such that the resulting theory is equivalent (but the dynamical objects do not need to be invariant).

defined. If this were not the case, the same law would appear different from two observers however close they are to one another. In a word then, (SP2) guarantees that the natural transformations of space-time do not turn physical facts into redundant structure.[24]

Now, since the bare configuration space violates the second facet of the dynamical matching condition, namely Earman's symmetry principle, we have therefore good reasons to think that configuration space cannot be "the theater in which physical laws act", to use Durr and Teufel's felicitous expression (2009, p. 44). More precisely, the bare configuration space cannot be the *stage* in which the physical laws act and whereby the wave function plays out. There is, so to speak, too much backstage for configuration space to be considered fundamental.

The importance of the mismatch between the natural structure of configuration space and the structure of the dynamical laws is recognized among others by Lewis (2004, 2013). As far as I can see, he puts the dilemma in its clearest terms: if we want to stick to wave function realism, we must either modify our intuitive conception of the nature of a fundamental physical space, or interpret the standard dynamical laws in an unusual way (2004, pp. 723-4). Lewis chooses the first horn and his own solution is to suggest that the configuration space is both $3N$ dimensional and three dimensional. On the one hand, the $3N$ dynamical parameters define the minimal number of independent parameters sufficient to describe the set of properties of the system at each time (that is, we need $3N$ coordinates to define completely $\psi(x_1, ..., x_{3N}, t)$ at any time t). Monton's concern (2006, p. 787) that Lewis leaves unclear the meaning of the "independence" between the parameters can be addressed as follows: the variables $(x_1, ..., x_{3N})$ are independent parameters of a differential equation, which means that we need at least $3N + 1$ parameters, time included, to account *completely* and *uniquely* for the *variations* of the function $\psi(x_1, ..., x_{3N}, t)$ given some initial value $\psi(x_1^0, ..., x_{3N}^0, t^0)$. On the other hand, the $3N$ dynamical parameters group into three independent directions, as defined by the coordinate transformations that leave the form of the dynamical equations invariant. That is, what Lewis means here is that the transformations leaving the dynamical relations between the ψ's must distinguish between three groups of N-coordinates.

Lewis' solution, however, fails for two main reasons, if we set aside the fact that he does not trace the origin of the dilemma back to the violation of the symmetry principle mentioned above. (i) The most crucial is that Lewis' space violates the condition (1), namely, that a fundamental space is defined (among others) by a minimal extensive structure. A space whose minimal structure is Euclidean can have only one topological dimension.[25] If the space has two genuinely distinct and equivalent dimensions, then it must have two distinct definitions of neighborhood and accordingly (since the space is

[24](SP2) is uncontroversial for most typical theories defined on Galilean and Minkowski space-times (e.g., Earman, 2004). However, the case of diffeomorphism invariance is much more problematic. It is not completely clear in this case what is exactly the physical (and thus dynamical) status of general covariance, the symmetry principle underlying diffeomorphism invariance. For more about this, see Earman (2004) and Brading and Castellani (2005). Note also that (SP1) and (SP2) do not undermine the possibility that some structure is emergent: for that, the dynamical equations must contain a symmetry subgroup (usually in some regime defined by relevant parameters) with match the symmetries of a subspace of the fundamental space.

[25]Monton (2006) makes a similar appeal to the inductive definition of topological dimension but fails to see the importance of this aspect.

Euclidean) two definitions of distances ($s_{3N}^2 = dx_1^2 + ... + dx_{3N}^2$ and $s_3^2 = dx_1^2 + dx_2^2 + dx_3^2$). There must be in other words two distinct minimal extensive structures and thus two distinct spaces (properly understood). But this makes Lewis' proposal inconsistent. The only solution left for Lewis' two-dimension proposal, provided that it presupposes two distinct senses of dimension for a single space, is that one of these senses of dimension is less fundamental, or emergent, or illusory. (ii) The second reason is that Lewis (2004, p. 727) interprets the spatial elements of configuration space not as bare points but as N-particle configurations, and this is rather counter-intuitive. The points p of the $3N$ configuration space have to be identified with sets of N elements $p = (p_1, ..., p_N)$. But these spatial elements are highly structured and non-identical, which makes the primitive structure of the fundamental space highly inhomogeneous.

Maybe then Lewis' failed attempt indicates that the $3N$ dimensional space must be structured differently. The best candidate which seems (at first sight) to satisfy Earman's symmetry principle is what I will call the "dressed configuration space", namely $\mathbb{R}^{3N}$ where we restrict the transformations on the space to $Gal(3,1)$ by grouping the coordinates into a triplet of N coordinates.[26] A translation $\vec{a} = (a_1, a_2, a_3)$ is for instance defined by:

$$(x_1, ..., x_{3N}) \rightarrow (x_1 + a_1, ..., x_{N+1} + a_2, ..., x_{2N+1} + a_3, ...) \tag{3}$$

Let me suggest a more refined construction: we can consider $\mathbb{R}^{3N}$ to be structured according to a three-dimensional subspace whose $Eucl(3)$ metric is induced as follows:

$$g_{ij}^3 = \sum_{a,b=1}^{3N} \frac{\partial x_a}{\partial y_i} \frac{\partial x_b}{\partial y_j} g_{ab}^{3N} \tag{4}$$

where g^{3N} is the natural Euclidean metric of configuration space. The $3N$ dimensional coordinates can be interpreted as $3N$ fields $(x_1(y_1, y_2, y_3), ..., x_{3N}(y_1, y_2, y_3))$, with (y_1, y_2, y_3) defining a coordinate system on the three dimensional subspace. We can also interpret this subspace as an infinite three dimensional static object living in $3N$ dimensions. By adjusting the dependence $\frac{\partial x_a}{\partial y_i}$ and how the fields transform given transformations on the three-dimensional subspace, we can obtain the correct $Gal(3,1)$ transformation in the $\mathbb{R}^{3N}$ which leaves the Hamiltonian invariant.

In both the brute and refined construction, the $\mathbb{R}^{3N}$ space has three "grooves" so to speak; that is, an absolute frame which defines three privileged directions along which the dynamical structure of the wave function remains invariant. As an example, we can imagine the case of two particles of identical mass moving and interacting along a one dimensional wire. The (typical and simplified) Hamiltonian involves one interaction term:

$$H_{12} = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + V_{12}(|x_1 - x_2|) \tag{5}$$

If we define two basis unit vectors $\vec{x}_1$ and $\vec{x}_2$ in the original frame, the privileged direction in

[26] The name "dressed configuration space" is meant to distinguish this space from what is typically called the "reduced state space". The latter corresponds to the quotient of the state space by the class of equivalent configurations.

configuration space for the wave function $\psi(x_1, x_2)$ is defined by the vector $\vec{a} = 1/\sqrt{2}(\vec{x}_1 + \vec{x}_2)$.

The problem with the dressed configuration space is that there is not a unique way of specifying the absolute frame (that is, the induced metric) independently from the wave function. For instance, we may rewrite the Schrödinger equation of the two-particle case with the new parameters $x_1^{'} = x_1 - x_2$ and $x_2^{'} = x_1 + x_2$. The Hamiltonian for the wave function $\psi(x_1^{'}, x_2^{'})$ takes the new form:

$$H_{12} = \frac{\partial^2}{\partial x_2^{'2}} + V_{12}(|x_1^{'}|) \tag{6}$$

This entails that the absolute frame now points into the direction $\vec{x}_1^{'}$ (and the privileged direction into the direction $\vec{x}_2^{'}$). In other words, the direction of the absolute frame is not fixed with respect to configuration space but only with respect to the dynamical facts encoded by the wave function. But it means that the induced metric and the Galilean structure $Gal(3,1)$ are therefore not *uniquely fixed* to configuration space itself. There is an infinite gauge freedom, that is, infinitely many indistinguishable ways of embedding the bare configuration space with a specific induced metric. And the condition (iii) of dynamical matching is therefore not completely fulfilled as in the case of the bare configuration space. But does it mean that we should step back and consider the three dimensional space as fundamental?

The primitive ontology approach does not seem to fare better. To cut a long story short, the wave function is non-separable on the three dimensional space. This means first, that asking about the value of the (multi-particle) wave function at a single point in three dimensional space is a meaningless question, and second, that the specification of physical properties at any location on this space leaves physical facts out. The three dimensional space thus violates the condition of physical completeness.

We have therefore reached the point where, on the standard proposals, neither wave function realism nor the primitive ontology approach provide a space which fulfills the four conditions of the minimal model. This seems to indicate that neither the configuration space nor the three dimensional space qualify as physically fundamental. One solution is to relax the conditions of the minimal model, or argue that some of them are more fundamental than the others. But, as I suggested in Sec. 3, the conditions which might serve to discriminate between the two approaches here (i.e. physical completeness and dynamical matching) seem to be on the same footing. Another route is to provide complementary conditions to discriminate between the different candidates for a fundamental physical space. Let me give a sketch of what these conditions could be.

(5) Epistemic access: The fundamental space does not commit us to specific structures which cannot be fully specified from empirical evidence (e.g. space-time holes, an outside of space-time).[27]

(6) Ontological parsimony: The fundamental space does not define metaphysical mon-

[27] For a recent discussion about the global properties of space-time such as hole-freeness, inextensibility, isotropy, global hyperbolicity, cf. Manchak (2011).

strosities (e.g., an infinite number of objects, non-local objects). It generally minimizes the number of fundamental objects.

(7) Spatio-temporal parsimony: the fundamental space minimizes the spatio-temporal surplus structure (e.g. absolute frame, structured spatial-elements).

(8) Autonomy: Physical facts tend to be attributed to the fundamental space instead of being assigned to objects (the latter tend to supervene or be grounded on these facts). That is, the fundamental space has maximally explanatory power and does not need extra additional structure.

(9) Simplicity: The fundamental space is maximally symmetric, provided these symmetries are compatible with the fundamental dynamics. This entails that the order and amount of spatio-temporal structure is maximally simple.

(10) Naturalness: The fundamental space (i) facilitates the connection to less fundamental spaces and has maximal unifying power; (ii) has a maximum of features compatible with our intuition of a physical space (e.g., a distinction between space and time, the form of a continuum, simply-connected paths, a notion of straightness, local isotropy and homogeneity, etc.); (iii) is minimally revisionary and ad-hoc.

Without any doubt, the three dimensional space satisfies better the condition of naturalness, and the configuration space the conditions of ontological parsimony (since the wave function is a non-local object on the three dimensional space) and autonomy. The others are more controversial and depend crucially on the versions of wave function realism and primitive ontology approach one favors. It is possible to evaluate each candidate space for each proposal on the basis of these complementary conditions. That being said, I don't think there is any decisive argument for why we should prefer some of these standards (5)-(10) over the others and eventually favor either configuration space realism or three dimensional space realism — hence my agnosticism about which of the two scores the best. The aim of this paper is more modest: I argue in Sec. 5 that the wave function field realism claim fails for the main candidate-spaces for wave function realism.[28]

5 Why the wave function is not a field

Besides the 'configuration space realism' claim, the standard wave function realism approach presupposes that the wave function qualifies as a physical field. Albert formulates

[28] Note that there is another alternative for the candidate-space for wave function realism. Goldstein et al. (2005) suggested in the context of Bohmian mechanics that the configuration space is to be constructed as a multi-connected space ${}^N\mathbb{R}^3$. See also Dürr et al. (2006); Maudlin (2013, p. 140); Lewis (2013, pp. 117 ff.). This space is defined as the $3N$ dimensional manifold built out of the set of (distinct) N-element subsets of $\mathbb{R}^3$ (i.e., ${}^N\mathbb{R}^3 := \{S \subseteq \mathbb{R}^3 || S| = N\}$), and a "point" in ${}^N\mathbb{R}^3$ (i.e. each configuration) takes the following form $p = \{(x_1, x_2, x_3), ..., (x_{3N-2}, x_{3N-1}, x_{3N})\}$. This sort of construction, however, do not seem to fare better than the bare and dressed configuration space. ${}^N\mathbb{R}^3$ naturally contains the three dimensional Euclidean metric and is thus implicitly embedded with the Galilean group $Gal(3,1)$ (i.e. we transform identically the N elements of each point of ${}^N\mathbb{R}^3$). But if this is all there is, the space has a three-dimensional topology and thus does not qualify as a $3N$ dimensional space (properly speaking), although the cardinality of the set of points defining this space is $3N$. We may construct a $3N$ dimensional Euclidean metric such that ${}^N\mathbb{R}^3$ is properly speaking $3N$ dimensional. But in this case, it seems that we face the same issue of dynamical matching that we faced with the bare and dressed configuration space.

this claim as follows:

> The sort of physical objects that wave functions *are*, on this way of thinking, are (plainly) *fields* — which is to say they are the sorts of objects whose states one specifies by specifying the values of some set of numbers at every point of the space where they live, the sorts of objects whose states one specifies (in this case) by specifying the value of two numbers (one of which is usually referred to as an amplitude, and the other as a phase) at every point in the universe's so-called configuration space. The values of the amplitudes and the phase are thought of (as with all fields) as intrinsic properties of the points in the configuration space with which they are associated. (1996, p. 278)

There are two aspects underlying the definition of fields here. First, Albert characterizes the mathematical definition of fields: namely, a field is defined as a map from a base space (e.g. $\mathbb{R}^{3N}$) to a target space (e.g., $\mathbb{C}$), where the base space defines the domain of the field, the target space defines the type of elements the field consists of, and the mapping defines the assignment of these elements to locations on the base space. Each field-element is assigned to one point of the base space in our case (and in the standard cases too). The type of these elements does not need be simple: the typical target space for quantum fields consists of an algebra of operators — these can be understood as determinables; while for classical fields, we only need numbers in order to define the field-elements — these can be directly understood as specifying the states of the field.

Second, Albert specifies the physical character of the field by claiming that it consists of a continuous net of intrinsic properties of the real and, as implicitly suggested, substantival configuration space. This characterization is controversial: it is indeed common to interpret fields as global particulars which are independent from the spatial points instead of intrinsic properties of the space (cf. French's discussion, 2013, in the context of wave function realism). Leaving this controversy aside, Albert's proposal has the merit of attempting to specify what it means for a field to be physical, and his proposal is ultimately that a physical field is something made out of local real properties of a substantival physical space. But since the wave function $\psi(x_1, ..., x_{3N}, t)$ is defined in a specific coordinate basis and that the field-elements are therefore assigned to coordinate values of points in the base-space, we may have good reasons to believe that Albert does not pay enough attention to the difference between the mathematical representation of fields and the object on which the representation is supposed to latch.[29] That is, Albert's characterization fails to specify enough what it means for a field to live in or inhabit a space.

Bell puts the wave function field realism claim in a slightly different way while discussing Bohmian mechanics:[30]

[29] This is the sort of distinction Maudlin pays attention to but he does not seem to provide further insights about what makes the representation physical (2007, p. 3169). The sort of gauge redundancy that he refers to in his (2013), namely the gauge freedom that we have in defining the global phase of the wave function, is quite unproblematic, at least for defining the physical character of the wave function as a field (cf. below).

[30] See also Bell (1987, pp. 134, 163, 204). There are also many other instances of this claim both among the proponents and opponents of wave function realism. For instance: "The wave function is a field in the

> Note that in the compound dynamical system [i.e. the wave function and the particle configuration] the wave is supposed to be just as 'real' and 'objective' as say the fields of classical Maxwell theory [...]. *No one can understand this theory until he is willing to think of ϕ as a real objective field rather than just a 'probability amplitude'.* (1987, p. 128)

What is interesting here is that Bell emphasizes that the field must be "objective". There are various ways to understand this feature, and the commonly adopted one is to require that the properties of the object are observer-independent, that is, independent from the spatio-temporal means by which we represent it. This sort of objectivity best goes under the name of coordinate-independence or (space-time) gauge-independence (e.g., Earman, 2004), and amounts to the fact that the specific labels or coordinates or ways of representing an object are not representative by themselves. The sets of equivalent labels must therefore be mapped to one another in such a way that the label-dependent representations refer to the same facts and properties of the object — hence the appeal to space-time symmetries.

Let us provide more details. First, the symmetry transformations on the coordinates of the object might modify the mathematical form of the object but they need at least to leave its dynamics invariant. This means that the initial mathematical form of the object and the transformed one are physically equivalent and thus refer to the same physical thing, even though they are expressed in two distinct coordinate systems. Second, the constraint of coordinate-independence is constitutive of the structure of fields in most of the standard cases. In technical terms, physical fields are defined as irreducible representations of a specific spatio-temporal symmetry group, which means that the structure of the field (in the target space) is uniquely specified given a set of transformation induced on the base space.[31] For instance, a scalar field and a vector field respectively transform under a Galilean transformation G as follows:

$$\phi(x) \to \phi'(x) = \phi(G^{-1}x) \tag{7}$$

$$\psi^{\mu}(x) \to {\psi'}^{\mu}(x) = G^{\mu}_{\nu}\psi^{\nu}(G^{-1}x) \tag{8}$$

where the μ, ν run over the indices of the Galilean four dimensional representation space. The same sort of rules applies to all the different sorts of fields we are accustomed to, whether we speak for instance of tensor and spinor fields.

If we sum up, there seems to be three conditions for a field to be physically real and thus to inhabit a physical space:

(a) The base space of the field qualifies as a physical space (whether we understand it relationally or substantivally).[32]

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sense that it is spread out completely over the space it inhabits, possessing values, amplitudes in particular, at each point in this space." (Ney, 2013, p. 169), and "I grant that the mathematical description of the wave function is as of a field evolving in $3N$-dimensional space." (Monton, 2013, p. 160).

[31] More specifically, the target space provides a representation space for symmetry transformations, say a N-dimensional vector space for a vector field. The representation of the symmetry transformation (i.e. a N-dimensional matrix) on this space is said to be irreducible if we cannot define a lower dimensional invariant representation subspace on which this representation acts.

[32] Note that the case of curved space-time is slightly more complicated since, on the standard definition,

(b) The field transforms adequately under a complete set of physical symmetries (e.g. its structure in the target space is preserved under symmetry transformations).

(c) The dynamics of the field and any observable facts about the field are invariant under these physical symmetries.

I take these conditions to be sufficient for the present purposes. But I will suggest in the concluding remarks that a field also needs to interact with other objects or itself in order to qualify as physical (i.e. an absolute object is not a physical object).

The argument against wave function field realism for the bare configuration space goes as follows. The wave function is supposedly a scalar field defined by $3N$ dynamical parameters and transforming under the Galilean group $Gal(3N, 1)$ on configuration space. In this case however, and as we have already seen above, the Hamiltonian is not invariant under the full set of coordinate transformations. If we translate the wave function five meters along the first coordinate of the $3N$ dimensional reference frame (for instance), we obtain two physically inequivalent wave-function histories (i.e. two distinct Hamiltonians). Now, given the full set of possible transformations, there are in fact (i) infinitely many inequivalent coordinate definitions of the wave function at each point (under the passive interpretation of the transformations) and (ii) infinitely many physically distinct wave-functions on configuration space (under the active interpretation). (i) implies a lack of objectivity and this prevents the wave function from being a physical field. Moreover, (ii) implies that there are many empirically inequivalent representations of the wave function, and this underdetermines the fact that there ought to be a unique physical field living in configuration space.

The argument for the dressed configuration space is slightly different. In this case, we have restricted (on the simplest construction) the set of possible transformations on the $3N + 1$ dimensional space-time as follows: there are $3N - 3$ coordinates along which the wave function is fixed (this defines the absolute frame) while three remaining directions along which it can be "dragged" or "moved" and for which we can define physically equivalent inertial frames. Let us use the two dimensional wave function $\psi(x_1, x_2)$ example of Sec. 4 with the absolute direction defined by $\vec{b} = 1/2(\vec{x}_1 - \vec{x}_2)$ to see where this leaves us. All the points of this wave function can be transformed along the direction orthogonal to the absolute direction (that is, along $\vec{a}$ as defined above). But this means that only the layers $L_c = \{\psi(x_1, x_2) | x_1 + x_2 = c\}$ with $c \in \mathbb{R}$ are transformed into one another, and thus that, properly speaking, these layers are the only parts of the field which are "free to move" without the physical situation being changed and thus which are "free" to inhabit the space. Note that if we could transform the field points in all the independent directions, then properly speaking each point of the field would be connected to any other through a symmetry transformation and thus be "free to move". In the dressed configuration space case, however, what properly lives in or inhabits the dressed configuration space is the set of "layers" defined by the $3N - 3$ subspace fixed by the absolute frame. The wave function is therefore a three dimensional $(3N - 3)$-parameter field (properly speaking)

the base space of tensor fields is defined by a product of tangent and cotangent vector spaces defined at a point p of the manifold.

on the dressed configuration space in the sense that each of its independent parameter consists of N coordinates. We may therefore conclude that the wave function does not have the geometrical character of a scalar field *in the* $3N$ *dimensional space* and thus does not qualify as a physical field on the dressed configuration according to the standard understanding.

The situation is actually even worse both for the bare and dressed configuration space cases. The Galilean transformations are supposed to leave the dynamics invariant at least along three directions in each case. But this is actually only correct for rotations and spatial and temporal translations. The Galilei transformations inducing a change of inertial frame along one or several of the three privileged direction do not leave the form of the Schrödinger equation intact. In the simple one-dimensional one-particle case, the (passive) transformation in the x direction ($x \to x' = x + vt$ and $t \to t' = t$) gives:

$$i\hbar\frac{\partial\psi'}{\partial t} - i\hbar v\frac{\partial\psi'}{\partial x} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi'}{\partial x^2} + \frac{\hbar^2}{mv}\frac{\partial^2\psi'}{\partial x\partial t} - \frac{\hbar^2}{2mv^2}\frac{\partial^2\psi'}{\partial t^2} + V'\psi' \tag{9}$$

In this case, the only way to make the dynamics invariant is to transform the wave function as $\psi \to \psi' = \exp(i(mvx - mv^2t/2)/\hbar)\psi$.[33] In the general case, we must enforce a similar local gauge transformation on the wave function — and a very unnatural indeed since the local gauge above $f(x,t) = \exp(i(mvx' - mv^2t/2)/\hbar$ is mass-dependent (!). In any case, all of this means that the wave function does not transform as a scalar field when we shift it from one inertial frame to another.

Perhaps we can absorb the additional gauge phase $f(x,t)$. There is indeed some ambiguity in the definition of the wave function in the abstract formalism (recall the beginning of Sec. 2). We may redefine the state vector (and more generally any vector, position vectors included) by multiplying it by an arbitrary phase such that the norm of the state vector does not change. It means that we can redefine the wave function $\psi(x,t)$ in the simple one-dimensional case above as $e^{i(\theta_{\psi(t)}+\theta_x)}\psi(x,t)$, where $\theta_{\psi(t)}$ and θ_x are two distinct arbitrary phases (and also distinct for different times and positions). Each vector is therefore more accurately represented by a "ray", that is, an equivalent class of vectors under multiplication by a phase. But in the standard formalism, we need to fix the phase for each vector before proceeding.[34]

The problem for us is that the initial freedom we have in choosing the various phases of the wave function is not sufficient to absorb the local phase obtained by performing all the possible Galilei transformations. We have, so to speak, one degree of freedom $\theta_{\psi(t)}$ for each time-dependent state vector and one degree of freedom θ_x for each position vector. But the Galilean gauge $f(x,t) = \exp(i(mvx - mv^2t/2)/\hbar$ depends on the product of v and x, and on the product of v and t (m and $\hbar$ are fixed). Since we can boost with different velocities, once we fix the initial gauge for a certain velocity, we cannot absorb anymore the ones obtained by boosting with a different velocity. Even the additional phase $\theta_{\psi'(t)}$

[33]E.g. see Peres (2002, p. 246). The unusual transformation of the wave function is noted among others by Belot (2012, p. 6) but he does not develop the full consequence of this fact, cf. below.

[34]We could work with rays if we redefine the theory in the projective Hilbert space.

of the transformed wave function $\psi'(x,t)$ does not suffice.[35]

The alternative is to deny the physical relevance of Galilean boosts — or, shall we say, their metaphysical relevance insofar as the boosts precisely prevent us from interpreting the wave function as a physical scalar field. The Galilean boosts (in particular) leave the probability distribution of the wave function invariant, that is, these transformations do not change the observable features of the wave function. This is more generally the case for all the relevant symmetries that leave the dynamics invariant, provided that they have a unitary (or anti-unitary) representation. In other words, the local phase that we obtain by boosting the $3N$ dimensional wave function is not observable. We could then deny that Galilean boosts are part of the set of physical symmetries of the configuration space, maintain that there is a privileged inertial frame, and keep the wave function as a scalar field.[36] A second option is to dismiss the wave function and take its absolute square as the real scalar field over configuration space. But since we are interested in the metaphysics of the wave function (and not in the probability of measurement outcomes it yields), the first option is to be preferred. But in this case, the wave function is not a physical field on the standards (a), (b), (c) presented above. We also saw that symmetry transformations were preventing us from interpreting the wave function as a physical field for both the bare and dressed configuration spaces. We must therefore conclude that the wave function is best interpreted as an absolute physical thing defining a fixed medium (recall we are not denying the 'wave function realism' claim (a) of Sec. 2). Moreover, the wave function does not really "live" in configuration space in the sense that it is not really free to move, i.e. we can freely "displace" it on the configuration space while keeping its dynamics invariant.

6 Conclusion: A series of appetizers

The main task of this paper has been to clarify the basis of the disagreement about the nature of the physical space underwriting the ontological status of the wave function. The main conclusion in that regard is that both the three dimensional space and the high dimensional configuration space underlying (respectively) the primitive ontology and wave function realism approaches do not fit well what we mean by a fundamental physical space. Instead of attempting to find out which of the two spaces scores the best, even among the variants we may devise for each approach, I showed that the wave function does not

[35] The equivalence class of wave functions defined from the rays in the Hilbert space is often taken as an argument that the wave function is not a field (cf. Goldstein and Zanghi, 2013, p. 97, n. 2, and Maudlin, 2013, pp. 131-5, for a related discussion). But this is in fact misleading. We could multiply any field by a local phase (provided it leaves the dynamics invariant) and redefine the field accordingly. Moreover, the arbitrary phase arising from the definition of vectors in the Hilbert space is not physically significant as compared to the one obtained by Galilei transformations. This last point is crucial since it dismisses the various remarks made about the fact that the initial arbitrary phase implies that there exists multiple representation of each physical possibility (e.g. Maudlin, 2013, pp. 131-5). These multiple representations are not genuinely distinct ones but merely conventional.

[36] Earman (1989, p. 50) gives a similar example for the case of Fourier's equation of heat conduction $\alpha\nabla^2 u(x,y,z,t) = \frac{\partial u}{\partial t}(x,y,z,t)$, with α the thermal diffusivity. If we assume that $u(x,y,z,t)$ is a scalar, the equation is not Galilean invariant. But it can be rewritten in a covariant form if we assume the equation to hold in a preferred inertial frame. Note however that in this case we can take the field to be a property of the material medium and the medium to pick out a privileged rest frame. In our case, the wave function is supposed to be the complete description of the system.

qualify as a physical field (in the standard sense) for both of the main configuration-space candidates for wave function realism. There are even more features of quantum theory which seem to support this claim and I only briefly sketch them in what follows as a matter of conclusion:

1. The wave functional: In quantum field theories, the wave function must be replaced by a wave functional over field configurations. The direct base space defined by field configurations has an infinite number of dimensions (if the theory is defined on the continuum), which suggests that the "fundamental space" over which the wave function lives does not satisfy the condition of minimal extensive structure (it is not clear whether the dimension and the metric are well-defined in this case, and whether we have any well-defined space-time symmetry group). In addition, the wave functional does not have the typical form of a physical field (although this is debatable).

2. Theoretic underdetermination and lack of naturalism: The wave function realism approach is mainly based on the Schrödinger picture. They are many other competitors which happen to be metaphysically relevant and for which the wave function is not fundamental (and thus certainly not a field). Furthermore, the Schrödinger picture is quite an unnaturalistic formalism and interpreting the ontology of quantum mechanics on this basis might be misguided.

3. Underdetermination of the projection space: The position basis from which the wave function is defined in the abstract formalism is not the only one. We could also consider the wave function in momentum space $\psi(p_1, ..., p_{3N}, t)$ and it is unclear whether the 'wave function field realism' claim holds in this case (one would have to argue in particular that momentum space is a physical base space).

4. Absolute character: Not only the wave function is best interpreted as an absolute object in the sense of being fixed to configuration space, as I have argued, but it is also absolute in the sense that it does not exchange energy or physically *interact* with any other physical object. This is particularly clear in the context of Bohmian mechanics since the hidden variables do not act on the wave function, and the wave function therefore does not satisfy the principle of action-reaction (Dürr et al., 1995; Belot, 2012). All the physical fields we are accustomed to either interact with other objects or with themselves.

Where does that leave us? I do not think that the failure of the 'wave function field realism' claim indicates that we should favor the primitive ontology approach either. There are other possible realistic interpretation of the wave function compatible with configuration space realism, which are likely to be specific to the solution to the measurement problem we favor (e.g. see Belot, 2012, for Bohmian mechanics). Given the absolute character of the wave function, we may also be led to interpret it in close analogy with the old-fashioned ethers of the 19th century — say, as a sort of a global absolute "blob" as it has been recently suggested.

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