More Cats than Dogs

Comparing Quantities in "Steps Towards a Constructive Nominalism": Three Problems

Devin Morse, Columbia University

Introduction: The Challenge

> We do not believe in abstract entities. No one supposes that abstract entities-classes, relations, properties, etc.- exist in space-time; but we mean more than this. We renounce them altogether…
>
> By renouncing abstract entities, we of course exclude all predicates which are not predicates of concrete individuals or explained in terms of predicates of concrete individuals. *Moreover, we reject any statement or definition - even one that explains some predicates of concrete individuals in terms of others - if it commits us to abstract entities...*
>
> *We shall, then, face problems of reducing predicates of abstract entities to predicates of concrete individuals…*
>
> (Goodman & Quine, "Steps Towards a Constructive Nominalism" (1947), 105-107, my italics)

This is the challenge Goodman & Quine (henceforth "G&Q") have set for themselves as nominalists: to give translations that do not quantify over abstract entities of certain sentences in our language.

In this paper I consider G&Q's attempt to translate the following sentence:

(**S**) There are more cats than dogs. (109)

I take attempts to translate sentences like these to hold particular interest, as do G&Q, as seen by the fact that their discussion of their method of translating **S** takes up more space than their discussion of any other translation. Of course, it isn't the subject matter that's interesting; we are not trying to figure out how to allocate resources at the pound. What we are looking for is a translation method for *any* sentence of the form "there are more x's s.t. φx then there are y's s.t. ψy" (where φ and ψ are formulas each with a free variable). Such sentences are completely pedestrian, the likes of which appear in ordinary discourse and have intuitively judged truth values and entailments. Yet **S** is also a sentence whose standard formalization - that the cardinality of the set of cats is greater than the cardinality of the set of dogs, that is, that there exists an injective function, but no bijective function, from the set of dogs to the set of cats - quantifies over an extravagant range of abstract entities. As such, **S** provides a prime test case for the nominalistic project G&Q undertake.

Unfortunately for their project, then, I argue that attempting to generalize G&Q's method of translation faces insuperable difficulties. This is particularly troubling given the confidence G&Q have in this method as the right approach (evidenced by their use this method later in their reconstruction of mathematics). I begin by describing G&Q's method of translation. I then consider three challenges to G&Q's approach, presented by cases of overlap, atomism, and inferential relationships.

1. Goodman and Quine's Proposal

I begin with some clarifications regarding the nature of G&Q's project. First, G&Q require that our translations be in first-order logic - "...nothing may be used but individual-variables, [and] quantification with respect to such variables…" (107) - unsurprisingly in line with Quine's way of determining ontological commitment as given in "On What There Is" (31-32). Second, I take G&Q to require, at least in "Steps", that translations preserve the truth values of the sentences translated (see 109: "...will be true if and only if…").[1] On whether they take their translations to be hermeneutic or revisionary - whether the translations must preserve *meaning* - I remain agnostic. With these clarifications out of the way, let us introduce G&Q's proposal.

G&Q rely on mereology for their translation of **S**, using "Part" to express the (primitive) parthood relation - I use the shorter "P". G&Q state that this parthood relation works as specified in "The Calculus of Individuals and Its Uses" ("Steps" 108, footnote) - that is, as described by classical mereology.[2] For ease of presentation I also make use of the other traditional predicates of mereology: "is a proper part of" expressed by "PP"[3], and "overlaps" expressed by "O" (where two objects do not overlap I say they are "discrete").[4] I also speak of sums of multiple objects.[5]

G&Q supplement mereology with the primitive predicate "is bigger than", expressed by "Bgr" and relating the spatial extension of individuals[6] (111, 113). G&Q do not provide any

---

[1] How do we specify the truth conditions of the natural language sentence translated such that we can check that it and its translation have the same truth conditions, without giving yet another formal translation? I leave this question aside, and rely - as G&Q implicitly do - on our intuitive ability to see when truth conditions do and do not match up.

[2] The following axioms by Cotnoir & Varzi (forthcoming) provide a nice reference for those unfamiliar with classical mereology:

Reflexivity: $\forall x \mathrm{P}xx$
Transitivity: $\forall xyz((\mathrm{P}xy \wedge \mathrm{P}yz) \rightarrow \mathrm{P}xz)$
Antisymmetry: $\forall xy((\mathrm{P}xy \wedge \mathrm{P}yx) \rightarrow x{=}y)$
Remainder: $\forall xy(\neg\mathrm{P}xy \rightarrow \exists z \forall w(\mathrm{P}wz \leftrightarrow (\mathrm{P}wx \wedge \neg\mathrm{O}wy)))$
Unrestricted Fusion (axiom schema): $\exists x\varphi x \rightarrow \exists z\mathrm{F}_{\varphi}z$ [where φ is a formula with a free variable and $\mathrm{F}_{\varphi}z =_{\mathrm{def}} \forall x(\varphi x \rightarrow \mathrm{P}xz) \wedge \forall y(\forall x(\varphi x \rightarrow \mathrm{P}xy) \rightarrow \mathrm{P}zy)$]

These are actually *approximations* of the mereology of "Calculus" which, oddly, makes reference to sets.

[3] $\mathrm{PP}xy =_{\mathrm{def}} \mathrm{P}xy \wedge x{\neq}y$

[4] $\mathrm{O}xy =_{\mathrm{def}} \exists z(\mathrm{P}zx \wedge \mathrm{P}zy)$

[5] "x is is the sum of $y_1, \ldots, y_n$" meaning $\forall w(\mathrm{O}wx \leftrightarrow (\mathrm{O}wy_1 \vee \ldots \vee \mathrm{O}wy_n))$

[6] It is a point G&Q do not note, but the fact that "Bgr" is a *primitive* predicate is important. It should not be analyzed as "occupies a greater number of cubic meters" or some such expression that compares quantities of units of spatial extension, for it is precisely such expressions that we are trying to translate. Indeed, any way of understanding size relations as relations between numbers of units is illegitimate in G&Q's nominalist framework. We should therefore

axioms for this relation, apparently content to rely on implicit intuitions about its properties. *We should* however axiomatize these intuitive notions, as without them G&Q's method does not work. There are two obvious candidates for our axioms - asymmetry and negative transitivity (which together imply transitivity):

(A1) $\forall xy(\mathrm{Bgr}\, xy \rightarrow \neg\mathrm{Bgr}\, yx)$

(A2) $\forall xyz((\neg\mathrm{Bgr}\, xy \wedge \neg\mathrm{Bgr}\, yz) \rightarrow \neg\mathrm{Bgr}\, xz)$

For ease I say "*x* is smaller than *y*" to mean that *y* is bigger than *x*, and "*x* is the same size as *y*" to mean that *x* is not bigger than *y* and *y* is not bigger than *x*. Importantly, "is the same size as" is an equivalence relation.

In addition, I introduce axioms concerning how "Bgr" interacts with parthood:

(A3) $\forall xy(\mathrm{Bgr}\, xy \rightarrow \exists z(\mathrm{P}zx \wedge \neg\mathrm{O}zy))$

(If x is bigger than y then x has a part discrete from y)

(A4) $\forall xy(\mathrm{PP}xy \rightarrow \mathrm{Bgr}\, yx)$

(If x is a proper part of y then y is bigger than x)[7]

define units of size by use of "Bgr" and related notions. For example, we might pick out some reference object *a* setting the size of a cubic meter, and then define "*x* is a cubic meter" as "*x* is the same size as *a*" and "*x* is *n* cubic meters" (where *n* is an integer) as "*x* is the same size as any sum of some mutually discrete individuals $y_1, y_2, \dots, y_n$ each of which is a cubic meter." (One might think that it is simpler to use "*x* is the sum of mutually discrete parts $y_1, y_2, \dots, y_n$ each of which is a cubic meter," but this might not work if the universe is atomistic and these atoms vary in size. In such a case [for n>1] *x* might be *n* cubic meters without having any parts that are a cubic meter.) It is simple to extend this to cases where *n* is not an integer.

[7] While it may seem like these two axioms should be derivable from a single more fundamental axiom, this is unfortunately not the case (as far as I can tell). It is clearer to see why if we consider not the "bigger than" relation, but the "at least as big as" relation ("*x* is at least as big as *y*" $=_{\mathrm{def}} \neg\mathrm{Bgr}\, yx$), and note that antisymmetry does not hold for this relation as is does for parthood.

One might worry whether these axioms depend for their plausibility on intuitions regarding mereological harmony[8]; one might be skeptical of such intuitions and wonder if denying them avoids some problems I identify in sections 2 and 3 of this paper. However, for my purposes I need no appeals to such intuitions: as we will see in section 4, **A1**-**A4** (with the addition of other axioms that I there specify) are necessary for G&Q's method to work *at all*, and as my concern is to show that G&Q's method does *not* work, the principle of charity requires me to assume whatever axioms allow it to work to the greatest extent that it can.

G&Q now define a predicate "is a bit" as "is just as big as the smallest animal among all cats and dogs" (110):

(**D**) $\mathrm{Bit}_{\mathrm{C,D}}x =_{\mathrm{def}} \forall y((\mathrm{C}y \vee \mathrm{D}y) \rightarrow \neg\mathrm{Bgr}\, xy) \;\&\; \exists z((\mathrm{C}z \vee \mathrm{D}z) \;\&\; \neg\mathrm{Bgr}\, zx)$[9]

Incidentally, this is one place where we can see how it is peculiar that G&Q have not given any axioms for "Bgr": we need to know that "is the same size as" is an equivalence relation - more specifically that it is euclidean - in order to ensure that bits as defined in **D** are all the same size *as eachother*.

So that I do not have to keep saying "a part of a cat which is a bit" and so forth, I follow G&Q and call *x* "a bit of" *y* when *x* is a bit and is a part of *y* (110). Although Q&G do not do so,

[8] For more on mereological harmony see Gilmore "Location and Mereology: 3. Mereological Harmony", The Stanford Encyclopedia of Philosophy.

[9] This formalization is mine, based on the formal definition of "Bit" Q&G give in the second half of their paper for different purposes (114).

it should be made clear that "Bit" is being defined in reference specifically to cats and dogs; I do so through my use of subscripts, following the schema:

(**DS**) $\mathrm{Bit}_{X1,\ldots,Xn}\, x =_{\mathrm{def}} \forall y((X_1 y \lor \ldots \lor X_n y) \rightarrow \neg \mathrm{Bgr}\, xy) \land \exists z((X_1 z \ldots \lor X_n z) \land \neg \mathrm{Bgr}\, zx)$

With these predicates introduced G&Q translate **S** as

(**T**) Every individual that contains a bit of each cat is bigger than some individual that contains a bit of each dog.

Formally:

(**TF**) $\forall x(\forall y(\mathrm{C}y \rightarrow \exists z(\mathrm{Bit}_{\mathrm{C,D}} z \land \mathrm{P}zy \land \mathrm{P}zx)) \rightarrow \exists w(\forall u(\mathrm{D}u \rightarrow \exists t(\mathrm{Bit}_{\mathrm{C,D}} t \land \mathrm{P}tu \land \mathrm{P}tw)) \land \mathrm{Bgr}$

$xw))$[10]

For the reader who finds it difficult to intuit what is being said here: imagine that we take a bit - a piece as big as the smallest thing that is either a cat or dog - from each and every cat and glom all these bits together, and then do likewise for dogs. There are more cats than dogs iff the thing we have glommed together from bits of each cat is bigger than the thing we've glommed together from bits of each dog. (More accurately: iff all the things we've glommed together that

[10] Again, my formalization, based on a formalization given in the second half of G&Q's paper (114). $\forall y(\mathrm{C}y \rightarrow \exists z(\mathrm{Bit}_{\mathrm{C}\lor\mathrm{D}} z \,\&\, z<y \,\&\, z<x))$ = "x is an individual containing a bit of each cat", and likewise for the similar clause in the second half of the formula with respect to dogs.

have at least one bit from each cat in them are bigger than at least one thing that has at least one bit from each dog.) Fortunately, this imagined procedure is a fiction; thanks to the axioms of classical mereology, specifically unrestricted fusion, we need not do anything so gruesome. The individuals that are sums of bits are spatially scattered, and the cats and dogs share their bits with these individuals without any loss of bodily integrity.

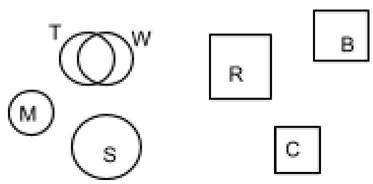
2. Problem I: Overlap

As G&Q note, while their method can handle many cases, and furthermore can be used in giving translations of other troublesome sentences (including those that seem to obviously make reference to abstract entities, such as "There are more age-classes than grade-classes in the White School", 110-111), it is not completely general - specifically, it assumes that for each predicate in our sentence the individuals fulfilling that predicate are mutually discrete[11] (110). G&Q do not describe the sort of difficult cases they have in mind, so it is worth giving some examples. Consider a model where the condition of mutual discreteness fails (Model 1 - size relations are as represented, and mereological overlap is represented by spatial overlap). There are four cats (represented by circles) - Whiskers, Thomas, Saidie, and Muffin - and three dogs (represented by squares) in the universe. Muffin is the smallest of the cats or dogs, and therefore x is a bit iff x is the same size as Muffin. Now the twist. Whiskers and Thomas are conjoined twins - they share a torso, and it so happens that their torso is as big as Muffin. All the other cats and dogs are discrete. *In this model it is not the case that every individual containing a bit of each cat is bigger than some individual containing a bit of each dog*, as there is an individual containing a

[11] There is, however, no problem with overlap between an individual fulfilling one of the predicates and an individual fulfilling another, e.g. "there are more human cells than humans" (110).

bit of each dog which contains three bits, and there is an individual containing a bit each of Whiskers, Thomas, Saidie, and Muffin which contains *only* three bits - *that individual which is the sum of one bit each from the four cats and whose bit each from Thomas and Whiskers is the same bit,* their shared torso.[12] Problems of this sort will arise not only when individuals fulfilling a predicate share bits, but when they have bits that overlap at all.

Model 1



G&Q state that there are ways to extend the method to handle such cases (as long as each individual fulfilling either predicate has a part that it shares with no other individual fulfilling that predicate) (110) but do not go into detail. Let us make some initial steps towards such extensions. First we add clauses to **T** specifying that each bit of a cat in the cat-bits containing individual does not overlap any bit of another cat where that bit is also in the cat-bits containing individual, and likewise for dogs. Formally:

(T.2) $\forall x(\forall y(\mathrm{C}y \rightarrow \exists z(\mathrm{Bit}_{\mathrm{C,D}}z \wedge \mathrm{P}zy \wedge \mathrm{P}zx \boldsymbol{\wedge \forall s((\mathrm{C}s \wedge s\neq y) \rightarrow \forall r((\mathrm{Bit}_{\mathrm{C,D}}r \wedge \mathrm{P}rs \wedge}$

$\boldsymbol{\mathrm{P}rx) \rightarrow}$

[12] In stating that this is a case where not every individual containing a bit of each cat is bigger than some individual containing a bit of each dog, I rely on a crucial (both here, and for G&Q's method to work in the first place) hidden premise that individuals that are sums of three bits are all the same size. This remains to be proven - and cannot be proven given the axioms specified so far. I show which axioms need to be added in the final section of this paper. For the moment, however, I will simply assume that this hidden premise is true. It is peculiar that G&Q, neither of whom can generally be accused of carelessness, would fail to notice that this is a concern.

$\neg \mathbf{O}\boldsymbol{zr}))))) \rightarrow \exists w(\forall u(\mathrm{D}u \rightarrow \exists t(\mathrm{Bit}_{\mathrm{C,D}}t \wedge \mathrm{P}tu \wedge \mathrm{P}tw \boldsymbol{\wedge} \boldsymbol{\forall q((\mathrm{D}q \wedge q \neq u) \rightarrow}$

$\boldsymbol{\forall p((\mathrm{Bit}_{\mathrm{C,D}}p \wedge \mathrm{P}pq}$

$\boldsymbol{\wedge \mathrm{P}pw) \rightarrow \neg \mathrm{O}tp)))) \wedge \mathrm{Bgr}\, xw))$

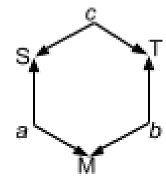
This covers all cases of overlap as long as each cat/dog has a bit that satisfies these new clauses - that is, a bit of that cat/dog for which each other cat/dog has a bit that the first bit does not overlap. Unfortunately we can't be sure of this - bits as we've defined them might be *too big*. Consider again our earlier model: as Muffin is the smallest of the cats or dogs, if Whisker's and Thomas' heads are each smaller than Muffin, then given **D** there will be no bit of Whiskers which does not overlap every bit of Thomas (and vice-versa).

This can be fixed. We redefine "Bit" as "the same size as the smallest maximal part of a cat or dog which does not overlap part of another cat or dog" (with "maximal" meaning "is not part of something else which satisfies this description"[13]). (In interest of space I do not give the formalizations of this or the following definitions.) We can also redefine "Bit" to handle other troublesome models where some number of cats or dogs do not have any parts that do not overlap parts of other cats or dogs, as long as every cat or dog has a part which it does not share with any other cat or dog: for example, where Muffin is the sum of parts *a* & *b*, Thomas is the sum of parts *b* & *c*, and Saidie is the sum of parts *c* & *a* (Model 2 - only the things that are parts of cats are pictured).[14]

[13] We need this addition because it may be that a part of a cat or dog which does not overlap a part of another cat or dog will itself have a proper part, which would be smaller (by **A2**) but would also satisfy this description. Saying that the part has to be maximal ensures that we don't therefore end up with our bits being atomic or, in an atomless world, undetermined in size.

[14] It should be clear at this point that I am continuing to speak of "cats" out of convenience - no collection of cats would have this mereological structure, or at the very least it would be deeply unpleasant if they did. The reader can

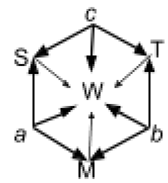
Model 2



Here we can redefine "Bit" as "the same size as the smallest maximal part of a cat or dog which only overlaps other parts of cats or dogs if it is part of those parts." We can, furthermore, give definitions of "Bit" to handle *some* models where cats or dogs are parts of other cats or dogs, for example those models in which the cats or dogs that contain other cats or dogs have parts which do not overlap the contained cats or dogs. However, there are cases where, as far as I can tell, no definition of "bit" can be given (as noted by G&Q (110)): for example, if we modify Model 2 so that the sum of *a*, *b*, and *c* is also a cat, say Whiskers (Model 3 - the lighter arrows indicate that according to classical mereology Muffin, Thomas, and Saidie would also be parts of Whiskers). While in some such models bits of the right size might still be available even if not definable, and so G&Q's method might still work, we will see in the second half of the next section that the possibility of models such as Model 3 show the *impossibility* in an atomistic universe of applying G&Q's method generally.

substitute some predicate for which it would make sense for objects satisfying that predicate to be organized in this way.

Model 3



3. Problem II: Atomism

Before considering the challenge just alluded to, there is a different problem with atomism I want to first consider. Suppose that the smallest cat is a mereological atom[15], and that this atom is smaller than any dog atom. *Then there are not any bits of dogs*, for a bit would need to be no bigger than this atomic cat, and the smallest parts of the dogs - their atoms (by **A4**) - are all bigger than a bit. Nothing as outlandish as atomic cats is needed for problems of this sort to arise - if the cats and dogs are all multi-atomed but the ratio of the sizes of their atoms is, say, √2 to 1 then there are no parts of cats that are the same size as any parts of dogs, and therefore there cannot be any bits of either one or the other (while irrational size ratios make this point most obvious, the problem is general: if the ratio of cat atom sizes to dog atoms sizes is *n*/*m*[16] [reduced to lowest terms] then the problem will arise if the smallest cat or dog is a cat and contains less than *m* atoms or if it is a dog and contains less than *n* atoms). Various other cases are imaginable (and indeed more plausible): different cats and dogs have different sizes of atoms, there are

[15] Where atom ("A") is defined: $\mathrm{A}x =_{\mathrm{def}} \forall y(\neg \mathrm{PP}yx)$

[16] This way of expressing size relations needs to be clarified - on the surface it seems to express size relations as ratios between numbers of units, and, as noted in an earlier footnote, in G&Q's system size relations have to be taken as primitive. One must therefore take "the ratio of sizes of cat atoms to dog atoms is *n*/*m*" to mean "if $x_1, x_2, \dots, x_m$ are mutually distinct cat atoms (all the same size) and $y_1, y_2, \dots, y_n$ are mutually distinct dog atoms (all the same size) then the sum of $x_1, x_2, \dots, x_m$ is the same size as the sum of $y_1, y_2, \dots, y_n$." Of course, strictly speaking, there is no nominalistically acceptable way to paraphrase "the ratio of sizes of cat atoms to dog atoms is √2 to 1", although it can be approximated arbitrarily closely, and we can of course easily say that there is no individual whose parts are all cat atoms that is the same size as some individual whose parts are all dogs atoms.

different sizes of atoms in individual cats and dogs, either cats or dogs but not both are atomless, etc. In all such cases the possibility arises that there is at least one cat or dog that does not have a part that is a bit, which would make **T.2** either trivially true or automatically false *regardless* of how many cats or dogs there are. This shows that *for Goodman and Quine's method to be generally applicable either each atom must not be bigger than any object*[17] *(which by **A1**, **A2**, and **A4** is true iff the universe is atomistic and all atoms are the same size), or the universe must not contain atoms.*

Returning to our original example, one might reply that even though in this case no dog can be divided into bits, the *spatial region* it occupies can be. Therefore we should substitute bits of *the spaces which the cats and dogs occupy* for bits of the cats and dogs. Of course *this* solution only works if *space* is atomless or its atoms are all the same size. It also requires the substantive[18] metaphysical commitment that *spatial regions* are objects in our ontology distinct from their occupants.[19]

These considerations show that we *may* not be able to apply G&Q's method generally in an atomistic universe. I now show that it is *impossible* to apply G&Q's method generally if the universe is atomistic - indeed, if there are any atoms in it whatsoever. Suppose *a* and *b* are amongst the atoms. We define the predicate "weird-cat" as follows: *x* is a weird cat iff $x = a$, $x =$

---

[17] $\forall x \forall y(\mathrm{A}x \rightarrow \neg \mathrm{Bgr}\, xy)$

[18] Pun intended. For a discussion of space-time substantivalism and dualist vs. monist views see Schaffer 2009, and also Gilmore "Location and Mereology", The Stanford Encyclopedia of Philosophy.

[19] The dualist substantivalist thesis also raises a puzzle particular to the "Bgr" relation: if I and the spatial region I occupy are distinct then there is a sum of the two of which the spatial region I occupy is a proper part. But then, by **A4**, this sum is bigger than the spatial region. But "bigger than" is supposed to relate spatial extension, so where is this sum whose spatial extension is greater than the spatial extension of the spatial region I occupy? (If you think there are ways to think about this in the case of specific individuals that makes it less perplexing, consider the entire universe [space along with the individuals in it] - the universe would be bigger than the entirety of space.)

b, or x = the sum of *a* and *b*. There are, therefore, three weird-cats. Suppose there are also two dogs in our universe. Can we translate the statement "there are more weird-cats than dogs" in the manner of **T.2** and get the correct truth-value? No - for there is *no way* to define "Bit" such that we can get a bit from each weird-cat that is discrete from a bit of some other weird-cat, as such discrete bits would have to be proper parts of atoms. Therefore, the general applicability of G&Q's method *requires* that the universe be non-atomistic. Of course, G&Q could deny that their method is supposed to be *this* generally applicable, and say that it should only be used when dealing with "normal" predicates (so not including weird-cats). Unfortunately, amongst the sentences their method are unable to handle are such consequences of classical mereology as "there are more individuals than atoms." Furthermore, this dodge fails to get out of the way of problem of variously sized atoms.

These challenges are troubling for two reasons. One for Goodman in particular is that his presentation of his own system is atomistic (Goodman 1956), so there is an inconsistency in his own overall corpus. It also is not clear whether a non-atomistic ontology is compatible with the finitism of "Steps". More importantly, the general applicability of G&Q's method depends on demonstrating that the universe has a certain structure - that it is non-atomistic, or at the very least that atoms are all the same size. I am inclined to think that these questions are empirical, in which case whether or not G&Q's method works depends on a scientific question that has not yet been decided - but even if it can be demonstrated *a priori* that the universe has the needed structure, it seems unreasonable that such a demonstration should be required for the general success of a translation method for something as simple as "there are more cats than dogs."

4. Problem III: Inferences

The final set of problems I consider are, I think, the most significant. A successful translation of a set of sentences should, I claim, license the inferences we make by preserving entailment relations amongst our sentences. That is, for each i s.t. $0<i<n$, the formal language sentences in $\{T_0, \ldots, T_{n\text{-}1}, T_n\}$ are successful translations of the natural language sentences in $\{S_0, \ldots, S_{n\text{-}1}, S_n\}$ only if $S_0, \ldots, S_{n\text{-}1}$ entails S_n iff $T_0, \ldots, T_{n\text{-}1}$ entails T_n. “$T_0, \ldots, T_{n\text{-}1}$ entails T_n” is simple logical entailment: every model where $T_0, \ldots, T_{n\text{-}1}$ is true is one where T_n is true. “$S_0, \ldots, S_{n\text{-}1}$ entails S_n” is a little trickier: I mean roughly that a competent speaker of the language would judge that every possible state of affairs where $S_0, \ldots, S_{n\text{-}1}$ are true is one where S_n is true. This might raise Quinean worries about appeals to analyticity (it resembles an appeal to “semantical rules” (Quine 1951, 31-34)) which in the interest of space I must set aside for now. For my own part, I take the entailment-preservation criterion to follow directly from the requirement that translations preserve truth conditions.

Do G&Q’s translation methods satisfy the entailment-preservation criterion? The inferences I consider are the following:

(**I1**) There are three cats.

There are two dogs.

Therefore there are more cats than dogs.

(**I2**) There are more cats than mice.

There are more mice than dogs.

Therefore there are more cats than dogs.

Before considering G&Q's translations, it's worth noting that the typical *platonistic* translations preserve entailments here quite nicely, and the following are valid inferences given the definitions of "=", ">", "3", and "2":

(I1.P) $|\{x\text{: } x \text{ is a cat}\}| = 3$

$|\{y\text{: } y \text{ is a dog}\}| = 2$

Therefore $|\{x\text{: } x \text{ is a cat}\}| > |\{y\text{: } y \text{ is a dog}\}|$

(I2.P) $|\{x\text{: } x \text{ is a cat}\}| > |\{y\text{: } y \text{ is a mouse}\}|$

$|\{x\text{: } x \text{ is a mouse}\}| > |\{y\text{: } y \text{ is a dog}\}|$

Therefore $|\{x\text{: } x \text{ is a cat}\}| > |\{y\text{: } y \text{ is a dog}\}|$

Unfortunately, G&Q's translations do not preserve these entailments given only the axioms we've specified.

Let us begin with **I1**. Employing G&Q's translations yields (for space and ease of reading I do not spell the translations out fully in first-order logic):

(I1.N) There are distinct objects *x*, *y*, and *z* such that anything is a cat iff it is *x* or *y* or *z* (108).

There are distinct objects *w* and *u* such that anything is a dog iff it is *w* or *u*.

> Therefore every individual that contains a bit of each cat is bigger than some individual that contains a bit of each dog. (i.e. **T**)

(I will assume from here on out that cats are mutually discrete, as are dogs, so as not to have to include the additional clauses in **T.2**.) Now obviously if there are three cats and two dogs (as appropriately translated) then the smallest individual containing a bit of each cat contains at least three bits and the smallest individual containing a bit of each dog contains at most two bits. Therefore *if* we had a theorem that individuals containing at least three discrete bits are bigger than individuals containing at most two discrete bits then **T** would be logically entailed by there being three cats and two dogs. *But the axioms I have given so far are compatible with a model in which there is an individual containing at least three bits and another individual containing at most two bits and the latter individual is bigger than the former.* As such it is contingent - at best an *empirical* generalization - that individuals consisting of three bits are bigger than individuals consisting of two bits. We must go out and *look* at such individuals to discover, lo and behold, that they are related in size this way, whereas we need not go out into the world to know that if there are three cats and two dogs then there are more cats than dogs.[20]

We can fix this. First note that *if* we could prove that all individuals that are the sums of two bits are the same size, then by **A1**, **A2**, and **A4** we could prove that individuals that contain at least three bits are bigger than individuals that are the sum of two bits[21] (and therefore bigger

[20] To be clear, the problem is not that the bits themselves might *change* size - indeed, we know that bits *are* all the same size - but that we haven't said enough about how the "Bgr" relation works.

[21] Proof: any individual that contains at least three bits has an individual that is the sum of two bits as a proper part, and so the former is bigger than the latter by **A4**, and by **A1** and **A2** if *x* is bigger than *y* and *z* isn't bigger than *y* then *x* is bigger than *z*, so if all individuals that are the sum of two bits are the same size and therefore none are bigger

than individuals that are at most two bits); the issue is that while we know that all bits are the same size, and we know by **A2** that individuals that are the sum of two bits are bigger than individuals that are single bits, we do not know what the size relations of these two-bit individuals are to *each other*, as we have nothing yet telling us what the size relation is between two objects based on the size relations of those objects' various parts. As such, we need additional axioms which give us this information. The following are good candidates (I only use **A7**, but I include all three for completeness[22]):

(**A5**) $\forall xyzw((\mathrm{Bgr}\, xy \wedge \mathrm{Bgr}\, zw \wedge \neg \mathrm{O}xz) \rightarrow \forall u((\mathrm{P}xu \wedge \mathrm{P}zu) \rightarrow \exists t(\mathrm{P}yt \wedge \mathrm{P}wt \wedge \mathrm{Bgr}\, ut)))$

(If x is bigger than y and z is bigger than w [and x and z are discrete[23]] then every individual with x and z as parts is bigger than some individual with y and w as parts)

(**A6**) $\forall xyzw((\mathrm{Bgr}\, xy \wedge \neg\mathrm{Bgr}\, zw \wedge \neg \mathrm{O}xw) \rightarrow \forall u((\mathrm{P}xu \wedge \mathrm{P}wu) \rightarrow \exists t(\mathrm{P}yt \wedge \mathrm{P}zt \wedge \mathrm{Bgr}\, ut)))$

(If x is bigger than y and z is not bigger than w [and x and w are discrete] then every individual with x and w as parts is bigger than some individual with y and z as parts)

(**A7**) $\forall xyzw((\neg\mathrm{Bgr}\, xy \wedge \neg\mathrm{Bgr}\, zw \wedge \neg \mathrm{O}yw) \rightarrow \forall u((\mathrm{P}yu \wedge \mathrm{P}wu) \rightarrow \exists t(\mathrm{P}xt \wedge \mathrm{P}zt \wedge \neg\mathrm{Bgr}\, tu)))$

---

than the others then any individual containing at least three bits is bigger than any individual that is the sum of two bits.

[22] As with **A3** & **A4** one might wonder whether **A5**-**A7** can be collapsed into one axiom. I do not see a straightforward means of doing so, especially as difference restrictions on overlap are made in each case.

[23] This clause is needed to a avoid cases such as, for example, *x* and *z* share most of their spatial extension and *x* is only slightly bigger than *y* and likewise for *z* and *w*, and *y* and *w* are discrete, in which case intuitively the sum of *x* and *z* might actually be *smaller* than the sum of *y* and *w*.

(If x is not bigger than y and z is not bigger than w [and y & w are discrete] then for every individual with y and w as parts there exists an individual with x and z as parts that is not bigger)

As all bits are the same size (by **A2** and the definition of "Bit"), **A7** implies that if two individuals are the same size then if a bit is added to each the resulting individuals are also the same size, and from this it can be easily seen how to demonstrate for any given number (we cannot, of course, prove for all numbers) that individuals that are the sum of that number of bits are all the same size. As already noted this entails that any individual containing three bits is bigger than any that is the sum of two. And this is what we needed in order to prove that if there are three cats and two dogs then every individual containing a bit of each cat is bigger than some individual containing a bit of each dog. So **I1.N** is, *with the appropriate axioms added*, a valid inference.[24]

I2 similarly turns out to be at best an empirical generalization given G&Q's translations without additional axioms. Now, it might not be obvious that G&Q's translations of the sentences in **I2** fail to yield a valid inference; after all, wouldn't it be translated as (again I do not bother spelling things out formally):

(**I2.N**) Every individual containing a bit of each cat is bigger than some individual containing a bit of each mouse.

Every individual containing a bit of each mouse is bigger than some individual

[24] Are axioms **A5-A7** ad hoc? I do not think so; although they are much more complicated than **A1**-**A4**, once they are understood I think it is intuitively clear that they describe properties we'd expect of a "bigger than" relation.

containing a bit of each dog.

Therefore every individual containing a bit of each cat is bigger than some individual containing a bit of each dog.

and isn't **I2.N** valid according to the transitivity of "Bgr"? No - remember, "bit" is defined in reference to the objects fulfilling the predicates being compared. **I2.N** should really be written, using the subscript system I introduced earlier:

(**I2.N***)Every individual containing a $\text{bit}_{C,M}$ of each cat is bigger than some individual containing a $\text{bit}_{C,M}$ of each mouse.

Every individual containing a $\text{bit}_{M,D}$ of each mouse is bigger than some individual containing a $\text{bit}_{M,D}$ of each dog.

Therefore every individual containing a $\text{bit}_{C,D}$ of each cat is bigger than some individual containing a $\text{bit}_{C,D}$ of each dog.

Which makes it immediately clear that **I2.N** only appeared to be valid only through equivocation in the use of "bit" - in the first sentence bits are the same size as the smallest of cats and mice, in the second of mice and dogs, and in the conclusion of cats and dogs.

Now, if we replace "$\text{bit}_{C,M}$", "$\text{bit}_{M,D}$", and "$\text{bit}_{C,D}$" all with "$\text{bit}_{M,C,D}$" - that is, we define "bit" throughout as the smallest of the mice, cats, or dogs - then the inference goes through. But this sort of move is unsatisfactory - it requires us to translate our sentences differently depending on what other sentences are in the inference we are considering. If we have translated the sentences in **I2** this way we are not, for example, able to add to our collection of sentences the translation of the sentence “there are more dogs than horses” and thereby get a set of sentences that logically entails the translation of the sentence “there are more mice than horses” without needing to retranslate every other sentence. Therefore if we are going to take this route we need a way to define “bit” across the board such that it is the same size as the smallest thing *period* - that is, the smallest of all the objects in the universe. But this requires that the universe be atomistic, and as noted earlier this presents insuperable problems for the general applicability of G&Q’s method.

Unlike with **I1.N**, I see no straightforward axioms in first-order logic to add to make **I2.N** valid. **A5**-**A7** won’t help us, as in order to apply them we would need to know the size relations between $\text{bits}_{C,M}$, $\text{bits}_{M,D}$, and $\text{bits}_{C,D}$, and how many cats, mice, and dogs there are (so we’d know how many of these various kinds of bits are in the various bits-containing individuals). Perhaps we can add an axiom *schema* to the effect that if every individual containing a $\text{bit}_{\varphi,\psi}$ of each φx is bigger than some individual containing a $\text{bit}_{\varphi,\psi}$ of each ψx, and every individual containing a $\text{bit}_{\psi,\chi}$ of each ψx is bigger than some individual containing a $\text{bit}_{\psi,\chi}$ of each χx, then every individual containing a $\text{bit}_{\varphi,\chi}$ of each φx is bigger than some individual containing a $\text{bit}_{\varphi,\chi}$ of each χx. But at this point I think we are firmly in the territory of *ad hoc* solutions.

If I am right about **I2.N**, then the translations generated by G&Q’s method fail the entailment-preservation criterion. I end the paper with a very brief discussion of two possible explanations as to why this does not seem to worry G&Q. First, as noted, one might be skeptical of that criterion, even if as I said the criterion follows from the requirement of truth-condition preservation. Perhaps one denies that nominalistic translation must have the same truth conditions as the sentence translated, that all that matters is that it serves whatever purposes we consider important (Quine in later work seems to think about translations this way: see "Word and Object" (1960), 196)). Well then, for my part, I think entailment-preservation is pretty important. Second, perhaps G&Q can bite the bullet, agree that successful translations must preserve entailments, that their translations do not produce entailments, and that their method *does* provide successful translations, and that therefore according to the entailment-preservation criterion *I* am mistaken about what qualifies as an entailment. Thus we are *not* licensed to treat **I2** as a logically valid inference. But, they might further say, this is not too worrisome, as the regularities of *our actual* world mean that it is an inference that will never lead us astray. And if one replies that we don't know for sure that such regularities hold, they might reply that nevertheless we need not abandon their method until we encounter a case where such regularities fail. Only *then* need we worry. Perhaps something like this response might arise from a rejection of the analytic-synthetic distinction (see Quine, 1951).

If one takes this final route I am not sure how to respond, except to say first that despite their assurances I feel losing **I2** as a valid inference is a high price to pay, and second that for my part I have more faith in the analytic-synthetic distinction and am less sure about the continuity of philosophy with science. Furthermore, even if we decide that inferences do not present a problem for G&Q's method, the problems presented by atoms and atomism still remain. As such, in one way or another, if not many, G&Q's method fails.[25]

[25] No animals were harmed in the writing of this paper.

Works Cited

Cotnoir, A.J & Varzi, A. *Mereology*. Oxford: Oxford University Press, Forthcoming.

Gilmore, Cody. "Location and Mereology." *The Stanford Encyclopedia of Philosophy.* Ed. Edward N. Zalta. 2017 ed. Stanford: Metaphysics Research Lab, Stanford University.

Goodman, N. "A World of Individuals." *The Problem of Universals: A Symposium*. Ed. J. M. Bochenski, A. Church, & N. Goodman. Notre Dame: University of Notre Dame Press, 1956. 13–31.

Goodman, N. & Quine, W.V. "Steps Toward a Constructive Nominalism." *The Journal of Symbolic Logic* 12.04 (1947): 105-22.

Leonard, H. S. & Goodman, N. "The Calculus of Individuals and Its Uses." *The Journal of Symbolic Logic* 5.02 (1940): 45-55.

Schaffer, J. "Spacetime the One Substance." *Philosophical Studies* 145 (2009): 131-148.

Quine, W. V. "On What There Is." *The Review of Metaphysics* 2.5 (1948): 21-38

---. "Main Trends in Recent Philosophy: Two Dogmas of Empiricism." *The Philosophical Review* 60.1 (1951): 20-43.

---. *Word and Object*. Boston: MIT Press, 1960.